Statistics

Lecture 9

Feb 19-8:47 AM

Class QZ 4

Use the chart below

x	$P(x)$
3	.1
5	.2
7	.4
9	.2
11	.1

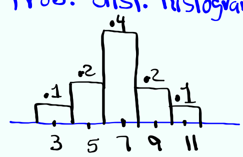
$\sum P(x) = 1$ ✓

$x \to L1$, $P(x) \to L2$

use $\boxed{\text{1-Var Stats}}$

with L1 & L2

1) Draw Prob. dist. histogram



find

2) $\mu = \bar{x} = 7$

3) $\sigma = \sigma_x = 2.191$

4) $\sigma^2 = 4.8 = \frac{24}{5}$

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$

$\boxed{4: \sigma_x}$ $\boxed{x^2}$ $\boxed{\text{Enter}}$

$\boxed{\text{Math}}$ $\boxed{\text{1:}\blacktriangleright\text{Frac}}$

$\boxed{\text{Enter}}$

$\mu = 7$, $\sigma \approx 2$

68% Range $\to \mu \pm \sigma \to$ $\boxed{5 \text{ to } 9}$

Usual Range $\to \mu \pm 2\sigma \to$ $\boxed{3 \text{ to } 11}$

"95% Range"

Oct 24-11:27 AM

Binomial Prob. dist: SG 16

1) n independents events (trials)

2) Each event has only two outcomes.

$P(\text{Success}) = p \qquad P(\text{Failure}) = q$

$p + q = 1$

$q = 1 - p$

3) p & q remain unchanged for all events.

4) $x \to$ # of Successes

$n - x \to$ # of failures

$$\boxed{P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}}$$

# of Combinations of x Successes in n trials

Oct 24-11:51 AM

Suppose $n = 5$ & $x = 2$

${}_5C_2 = 10$

5 [Math] [→] PRB [↓] nCr 2 [enter]

10 ways to have 2 Successes in 5 trials.

Find ${}_{12}C_5 = 792$

12 [Math] [→] PRB [↓] nCr 5 [enter]

in 12 trials, there are 792 different Combinations to have 5 Successes.

CA Lotto, choose 5 numbers from 1 to 5.

Combinations ${}_{50}C_5 = \boxed{2,118,760}$

Oct 24-12:04 PM

Consider a binomial Prob. dist. with $n=5$ and $p=.8$.

$q = 1 - p = 1 - .8 = \boxed{.2}$

P(exactly 3 Successes) =

$P(x=3) = {}_5C_3 \cdot (.8)^3 (.2)^2$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$= 10 \cdot (.8)^3 \cdot (.2)^2$

$= \boxed{.205}$

Oct 24-12:11 PM

You flip a fair coin 10 times.

Success is to land tails

$n = 10$

$p = .5$

$q = .5$

P(exactly 6 tails)

$P(x=6) = {}_{10}C_6 \cdot (.5)^6 \cdot (.5)^4$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$= 210 \cdot (.5)^6 \cdot (.5)^4 = \boxed{.205}$

Oct 24-12:17 PM

You are taking a multiple-choice exam.

There are $\underbrace{\text{20 questions}}_{n=20}$, each question has $\boxed{\text{4 choices}}$ with $\boxed{\text{one correct choice}}$. $p=\frac{1}{4}=.25$

$q=\frac{3}{4}=.75$

You are making random guesses.

P(guess exactly 8 correct ans)

$P(x=8) = {}_{20}C_8 \cdot (.25)^8 (.75)^{12}$

${}_nC_x \cdot p^x \cdot q^{n-x}$

$=125970(.25)^8(.75)^{12} = \boxed{.061}$

P(guess all Correctly)

$P(x=20) = {}_{20}C_{20} \cdot (.25)^{20} (.75)^0$

$= \boxed{9.1 \times 10^{-13}}$

Oct 24-12:22 PM

Using TI

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ $\boxed{\downarrow}$ $\boxed{\text{binompdf}}$(20 , .25, 20)

n P x

Trials: 20

P: .25

x: 20

$\boxed{\text{Paste}}$ $\boxed{\text{Enter}}$

No Menu

Your work

$P(x=20) = \text{binompdf}(20, .25, 20)$

$= \boxed{9.1 \times 10^{-13}}$

Oct 24-12:30 PM

P(guess Correctly on exactly 5 questions)

$P(x=5) = \text{binompdf}(20, .25, 5)$

$= \boxed{.202}$

P(guess wrong on all questions)

$P(x=0) = \text{binompdf}(20, .25, 0) = \boxed{.003}$

Oct 24-12:34 PM

Consider tossing a fair coin 100 times.

Success is to land tails

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $np = 50$ 5) $npq = 25$ 6) $\sqrt{npq}$ $= \sqrt{25} = 5$

7) P(lands tails 55 times)

$P(x=55) = \text{binompdf}(100, .5, 55) = \boxed{.048}$

8) P(lands tails 50 times) =

$P(x=50) = \text{binompdf}(100, .5, 50) = \boxed{.080}$

Oct 24-12:38 PM

9) P($\underbrace{\text{at most 60}}$ tails)

$P(x \le 60) = P(x=60) + P(x=59) + P(x=58) + \cdots + P(x=0)$

$= \text{binomcdf}(100, .5, 60)$

$= \boxed{.982}$

P($\underbrace{\text{fewer than 60}}$ tails)

$P(x < 60) = P(x \le 59) = \text{binomcdf}(100, .5, 59)$

$= \boxed{.972}$

Oct 24-12:44 PM

Consider binomial dist with $n = 150$ & $P = .6$

1) $q = 1 - p = \boxed{.4}$

2) $np = 150(.6) = \boxed{90}$

3) $npq = 150(.6)(.4) = \boxed{36}$

4) $\sqrt{npq} = \sqrt{36} = \boxed{6}$

Let x be # of Successes

5) $P(x = 100) = \text{binompdf}(150, .6, 100) = \boxed{.017}$

6) $P(x \le 100) = \text{binomcdf}(150, .6, 100) = \boxed{.961}$

7) $P(x \ge 85) = 1 - P(x \le 84)$

Total Prob.

we don't want 84 | 85 we want

$= 1 - \text{binomcdf}(150, .6, 84)$

$= \boxed{.821}$

Oct 24-1:01 PM

Working with Binomial Prob. dist:

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard deviation $\sigma = \sqrt{\sigma^2}$

Oct 24-1:10 PM

400 new born babies were randomly Selected.

Success is to have a girl.

1) $n = 400$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np = \boxed{200}$ 5) $\sigma^2 = npq = \boxed{100}$ 6) $\sigma = \sqrt{\sigma^2} = \boxed{10}$

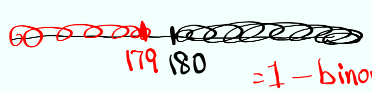
7) P(have $\underbrace{\text{at most}}$ 210 girls)

$P(x \le 210) = \text{binomcdf}(400, .5, 210)$

$= \boxed{.853}$

8) P(have $\underbrace{\text{at least}}$ 180 girls)

$P(x \ge 180) = 1 - P(x \le 179)$

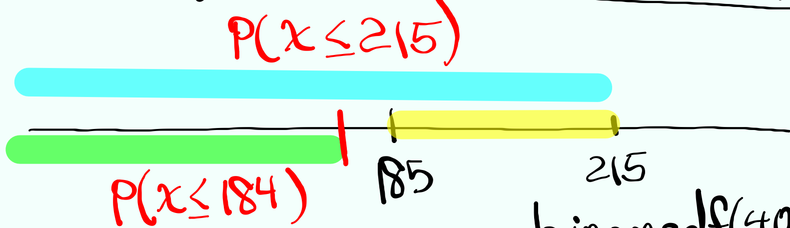


$= 1 - \text{binomcdf}(400, .5, 179)$ $= \boxed{.980}$

Oct 24-1:12 PM

P(# of girls is between 185 and 215, inclusive)

$$P(185 \le x \le 215) = P(\overbrace{x \le 215}^{\text{Keep}}) - P(\underbrace{x \le 184}_{\text{Reduce by 1}})$$



= binomcdf(400,.5,215) − binomcdf(400,.5,184)

= $\boxed{.879}$

Oct 24-1:19 PM

I randomly selected 400 voters.
Prob. that any voter votes yes on Prop 50 is .8.

Success is to vote yes.

1) $n=400$ 2) $p=.8$ 3) $q=.2$

4) $\mu=np=320$ 5) $\sigma^2=npq=64$ 6) $\sigma=\sqrt{\sigma^2}=8$

7) P(# of yes is between 310 and 330, inclusive)

$$P(310 \le x \le 330) = P(\overbrace{x \le 330}^{\text{Keep}}) - P(\underbrace{x \le 309}_{\text{Reduce by 1}})$$

= binomcdf(400,.8,330) − binomcdf(400,.8,309)

= $\boxed{.811}$

SG 16 ✓

Oct 24-1:24 PM

Prob. dist. with Continuous Random Variable

1) Uniform Prob. dist

2) Standard normal Prob. dist.

3) Normal prob. dist

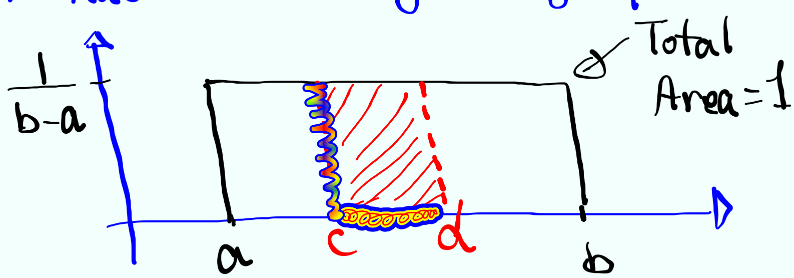
4) Central Limit theorom

5) Application

Oct 24-1:37 PM

Uniform prob. dist.

It is for all values from a to b and it has a rectangular graph

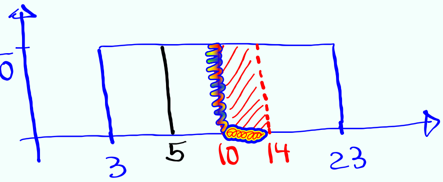


$P(x=c)=0$ $\qquad$ $P(c<x<d)=(d-c)\cdot\frac{1}{b-a}$

Oct 24-1:51 PM

Consider a uniform prob. dist. for all values from 3 to 23.

$\frac{1}{23-3} = \frac{1}{20}$



$P(x=5) = \boxed{0}$

↑ Line

$P(10 < x < 14)$

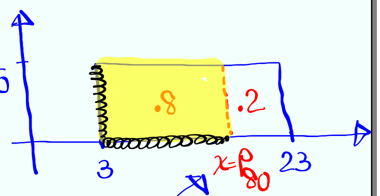
$= (14-10) \cdot \frac{1}{20} = \frac{4}{20} = \frac{1}{5} = .2$

Find $x = P_{80}$

80% below

20% above

$\frac{1}{20}$



$(x-3) \cdot \frac{1}{20} = .8$

$x - 3 = 20(.8)$

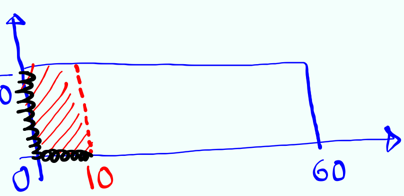
$x = 3 + 16$

$\boxed{x = 19}$

Oct 24-1:55 PM

Wait time to be seen at DMV has a uniform Prob. dist with up to 60 Min.

$\frac{1}{60}$



P(wait time is less than 10 mins.)

$P(x < 10) = (10-0) \cdot \frac{1}{60} = \frac{10}{60} = \boxed{\frac{1}{6}}$

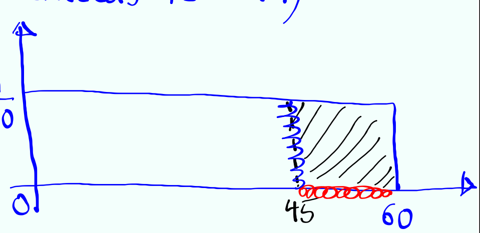
P(wait time exceeds 45 mins.)

$P(x > 45)$

$= (60-45) \cdot \frac{1}{60}$

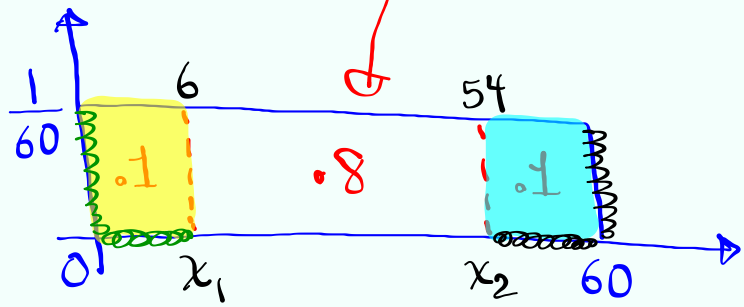
$= \frac{15}{60} = \boxed{\frac{1}{4}}$

$\frac{1}{60}$



Oct 24-2:02 PM

Find two times, rounded to whole #s that separate the $\boxed{\text{middle } 80\%}$ from the rest.



$(x_1 - 0) \cdot \frac{1}{60} = .1$

$x_1 = 60(.1)$

$\boxed{x_1 = 6}$

6

$(60 - x_2) \cdot \frac{1}{60} = .1$

$60 - x_2 = 60(.6)$

$\vdots$ $\boxed{x = 54}$

Oct 24-2:08 PM

Consider a binomial Prob. dist. with $n = 120$, $p = \frac{1}{3}$.

1) $q = \frac{2}{3}$

2) $\mu = 120 \cdot \frac{1}{3} = 40$

3) $\sigma^2 = 120 \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{80}{3}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{80}{3}} \approx 5$

68% Range $\mu \pm \sigma \rightarrow \boxed{35 \text{ to } 45}$

$P(x < 50) = P(x \le 49) = \text{binomcdf}(120, \frac{1}{3}, 49) = \boxed{.965}$

$P(35 \le x \le 45) = \text{binomcdf}(120, 1/3, 45) - \text{binomcdf}(120, 1/3, 34)$

$\boxed{.713}$

Oct 24-2:14 PM

Pay \$2 draw 1 Card, if its red Ace,
I give You \$50.
Expected Value for player.

Net	P(Net)	
50 − 2	2/52	Red Ace
0 − 2	50/52	$\overline{\text{Red Ace}}$

$E.V. = \mu = \overline{x}$

\$ −.08

≃ −8 ¢

Oct 24-2:23 PM